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Dust Sheared Flow Driven Instability of Dust Drift Waves in a Nonuniform Magnetoplasma

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We investigate instability of dust drift waves in a nonuniform dusty magnetoplasma containing transverse sheared plasma flow that is produced by a nonuniform electric field. By using Boltzmann distributed electrons and ions, Poisson's equation, as well as the dust continuity equation with perpendicular guiding center dust drift speed, we derive an eigenvalue equation, which strongly depends on the profiles of dust sheared flow and dust density gradient. The eigenvalue equation is analytically solved to obtain expressions for the growth rate and threshold of a convective instability arising from resonant interactions between the dust drift waves and sheared flows. The result may be relevant to the understanding of short wavelength (in comparison with the ion gyroradius) electrostatic fluctuations in magnetized plasmas of Saturn rings and in cometary tails.

KEY WORDS: dust plasma; plasma instability.

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1. INTRODUCTION

Recently, there has been growing interest (Shukla, 2003a; Shukla and Mamun, 2002; Verheest, 2000) in studying collective processes in dusty plasmas whose constituents are electrons, ions and electrically charged extremely heavy dust grains. The latter are usually negatively charged, with a large number of electrons on each dust grain, they are of micron or submicron size, and they have a mass that is significantly larger than the positive ion mass. The negative charge of the dust particles could be due to different processes, such as collection of electrons from surrounding plasmas, ultraviolet ray irradiation, sputtering of energetic ions, etc. Dusty plasmas are found in space environments (Goertz, 1989; Mendis, 2002;

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Shukla, 2003b), such as the lower ionosphere of the Earth, planetary atmospheres, asteroid zones, nebulae, cometary comae and tails, as well as in a variety of low-temperature plasma devices (Bouchoule, 1999; Thomas *et al.*, 2003; Winter, 2000).

Dusty plasmas in Saturn rings and cometary plasmas are magnetized. In magnetized plasmas, charged dust as well as electrons and ions can be magnetized. However, there are situations when the dust grains are magnetized, while electrons and ions obey Boltzmann distributions in the electrostatic field $\mathbf{E} = -\nabla\phi$, where ϕ is the electrostatic potential. In such a circumstance, there exists the possibility of stable dust drift waves (DDWs) (Shukla *et al.*, 1993) in a nonuniform dusty magnetoplasma.

In this paper, we show that equilibrium transverse sheared dust flow can amplify the DDWs. For this purpose, we derive the eigenvalue equation for the DDWs by using Boltzmann distributed electrons and ions, Poisson's equation, and the dust continuity equation in which the perpendicular (to the external magnetic field $\hat{\mathbf{z}}B_0$, where $\hat{\mathbf{z}}$ is the unit vector along the z axis and B_0 is the strength of the external magnetic field) component of the dust fluid velocity is the sum of the $\mathbf{E} \times \mathbf{B}_0$ and polarization drifts. The eigenvalue equation is analyzed to obtain the eigenfunctions, threshold and the growth rate of a Rayleigh instability caused by transverse sheared dust flows generated by an inhomogeneous dc electric field in plasmas.

2. GOVERNING EQUATIONS

Let us consider a nonuniform dusty plasma composed of electrons, ions, and negatively charged dust grains. At equilibrium, we have $n_{i0}(x) = n_{e0}(x) + Z_d n_{d0}(x)$, where n_{j0} is the unperturbed number density of the particle species j (j equals e for electrons, i for ions, and d for dust grains) and Z_d is the number of electrons residing on the dust. In the equilibrium, we also have an equilibrium transverse (with respect to $\hat{\mathbf{z}}$) dust flow speed $\vec{v}_0 = \hat{\mathbf{y}}v_0(x) \equiv -\hat{\mathbf{y}}(c/B_0)\partial\phi_0/\partial x$, which is nonuniform. Here, c is the speed of light in vacuum and the equilibrium electric field is $\mathbf{E}_0 = -\hat{\mathbf{x}}\partial\phi_0/\partial x$. The unit vectors along the x and y axes are denoted by $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$, respectively.

We consider electrostatic perturbations whose wavelengths lie between the electron and ion gyroradii and whose frequencies are much smaller that the dust gyrofrequency $\omega_{cd} = Z_d e B_0/m_d c$, where m_d is the dust mass. The electron and ion density perturbations are then given by, respectively,

$$n_{e1} = n_{e0}\frac{e\phi}{T_e}, \tag{1}$$

and

$$n_{i1} = -n_{i0}\frac{e\phi}{T_i}, \tag{2}$$

where e is the magnitude of the electron charge and $T_e(T_i)$ is the electron (ion) temperature.

The dust dynamics is governed by the dust continuity equation

$$\frac{\partial n_{d1}}{\partial t} + \nabla \cdot (n_{d0}\mathbf{v}_{d\perp}) = 0, \tag{3}$$

where $n_{d1}(\ll n_{d0})$ is a small perturbation in the dust number density and the perpendicular (to $\hat{\mathbf{z}}$) component of the dust fluid velocity in drift approximation (viz. $|\partial/\partial t| \ll \omega_{cd}$) is

$$\mathbf{v}_{d\perp} = \frac{c}{B_0}\hat{\mathbf{z}} \times \nabla_\perp\phi - \frac{c}{B_0\omega_{c_d}}\left(\frac{\partial}{\partial t} + ik_y v_0\right)\nabla_\perp\phi. \tag{4}$$

Equations (1)–(4) are closed by using Poisson's equation

$$\nabla^2\phi = 4\pi e(n_{e1} - n_{i1} + Z_d n_{d1}). \tag{5}$$

Combining (1)–(4) and supposing that $\phi = \varphi(x)\exp(-i\omega t + ik_y y)$, we obtain the eigenvalue equation

$$\left(\frac{\partial^2}{\partial x^2} - \bar{k}_d^2\right)\varphi + \frac{k_y}{(\omega - k_y v_0)}\frac{\alpha - 1}{\alpha}\left[\omega_{cd}\frac{\partial}{\partial x}\left(\frac{n_{d0}}{B_0}\right) - v_0''(x)\right]\varphi = 0, \tag{6}$$

where $\bar{k}_d^2 = k_y^2 + k_d^2/\alpha$, $k_d^2 = 4\pi n_{i0}e^2(1+\sigma)/T_i$, $\sigma = n_{e0}T_i/n_{i0}T_e$, $\alpha = 1 + \omega_{pd}^2/\omega_{cd}^2$, and $\omega_{pd} = (4\pi n_{d0}Z_d^2e^2/m_d)^{1/2}$ is the dust plasma frequency.

3. INSTABILITY ANALYSIS

The necessary condition(viz. the Rayleigh-type) for the instability can be readily found by multiplying Eq. (6) by a complex conjugate φ^*, assuming the complex frequency $\omega = \omega_r + i\omega_i$, and integrating the resultant equation perpendicular to the flow direction, i.e. in the x direction. We obtain

$$v_0''(x) - \omega_{c_d}\frac{\partial}{\partial x}\left(\frac{n_{d_0}}{B_0}\right) = 0 \tag{7}$$

where the asterisk stands for the complex conjugate. Thus, an absolute instability is possible if at any point x_c in the transverse direction (perpendicular to the direction of the flow), condition (7) is satisfied. One should notice that the presence of the dust density gradient modifies the standard condition requiring the existence of an inflection point in the flow profile.

Further linear stability shall be discussed in detail for some specific profiles of the shear plasma flow and dust particles density. We suppose the plasma flow profile is described by the following well behaved analytic function

$$v_0(x) = u + a\,\kappa \tanh(\kappa x), \tag{8}$$

where κ^{-1} defines the characteristic width (slope) of the shear flow. We now proceed calculating the appropriate derivatives of $v_0(x)$ and introducing a new variable $\eta = \tanh(\kappa x)$, and by assuming a physically interesting case when $u = \omega/k_y$ and a is constant. We rewrite Eq. (6) in the form

$$(1-\eta^2)\frac{d^2\varphi}{d\eta^2} - 2\eta\frac{d\varphi}{d\eta} + \left[2\left(\frac{\alpha-1}{\alpha}\right) - \frac{\bar{k}_d^2+1}{\kappa^2}\frac{1}{1-\eta^2} + \frac{1}{\kappa^2}\frac{1}{1-\eta^2}\right.$$
$$\left. - \left(\frac{\alpha-1}{\alpha}\right)\frac{\omega_{c_d}}{B_0}\frac{1}{a\kappa^2}\frac{1}{\eta}\frac{d\,n_{d_0}}{d\eta}\right]\hat{\varphi} = 0. \tag{9}$$

Obviously, for n_{d_0} satisfying

$$n_{d_0} = \frac{a\alpha}{2(1-\alpha)}\frac{B_0}{\omega_{c_d}}\left(1-\eta^2\right) \equiv \frac{a\alpha}{2(1-\alpha)}\frac{B_0}{\omega_{c_d}}\left[1-\tanh^2(\kappa x)\right] \tag{10}$$

The solution of Eq. (9) become Legendre functions of the degree 1 and of the order $\nu = (\bar{k}_d^2+1)^{1/2}/\kappa$, i.e.

$$\varphi(\eta) = C_1 P_1^{\nu}(\eta) + C_2 Q_1^{\nu}(\eta) \tag{11}$$

where

$$P_1^{\nu}(\eta) = \frac{\eta-\nu}{\Gamma(2-\nu)}\left(\frac{\eta+1}{\eta-1}\right)^{\nu/2},$$
$$Q_1^{\nu}(\eta) = -\frac{\pi}{2\Gamma(2-\nu)}\frac{\exp(\mathrm{i}\nu\pi)}{\sin(\nu\pi)}\left[(\eta+\nu)\left(\frac{\eta-1}{\eta+1}\right)^{\nu/2}\right.$$
$$\left. - (\eta+\nu-2)\left(\frac{\eta+1}{\eta-1}\right)^{\nu/2}\right].$$

The only localized solutions for $\eta \to \pm 1$, i.e., for $x \to \pm\infty$ are possible if

$$\nu^2 \equiv \frac{1+\bar{k}_d^2}{\kappa^2} = 1. \tag{12}$$

The DDW becomes convectively unstable (growing in space) because of the resonant interaction with the flow, provided that the wavenumber satisfies the condition (Vranjes and Jovanovic, 1996; Vranjes and Tanaka, 2002; Vranjes *et al.*, 2001)

$$k_y > \left(\kappa^2 - \frac{k_d^2}{\alpha} - 1\right)^{1/2}. \tag{13}$$

In that case, assuming a small deviation δk_y of the stable value of the wavenumber k_y, and letting $\omega = \omega_r + i\gamma$, we obtain the growth rate

$$\gamma = \frac{4\delta k_y \left[\kappa^2 - \left(k_d^2/\alpha\right) - 1\right]^2}{\pi\kappa\left[1 - 2\kappa^2(\alpha - 1)/\alpha\right]} \left|\frac{dv_0}{dx}\right|_{x=0}, \tag{14}$$

which is positive definite if $\kappa^2 < \alpha/2(\alpha - 1)$. Now, for the profiles of $v_0(x)$ and n_{α_0} given by Eqs. (8) and (10), using the Rayleigh condition (7), we obtain the following connection between the flow width κ^{-1} and the critical points for the instability

$$\tanh(\kappa\, x_c) = \sqrt{1 + \frac{\alpha}{2a\kappa^2(\alpha - 1)}}. \tag{15}$$

Thus, an absolute instability at the point x_c for modes with wavenumbers satisfying the condition (14) can be expected. Furthermore, it follows from Eq. (15) that there appears a limiting value of $\kappa_L \approx 0.71$ above which the critical point x_c becomes complex, i.e. the Rayleigh type instability vanishes.

4. CONCLUSIONS

In this paper, we have considered instability of low-frequency (in comparison with the dust gyrofrequency), short wavelength (in comparison with the ion gyroradius) dust drift waves in a nonuniform containing dust density gradient and transverse dust sheared flow in a magnetized dusty plasma. It has been found that free energy stored in dust sheared flow can be coupled to DDWs whose wavelength is smaller than $[\kappa^2 - (k_d^2/\alpha) - 1]^{-1/2}$. Furthermore, the growth rate of the present Rayleigh-type instability strongly depends on the magnitude of dv_0/dx at $x = 0$ as well as on κ^2 which should be smaller than $\alpha/2(\alpha - 1)$. The dust sheared flow driven electrostatic fluctuations can account for low-frequency disturbances in Saturn rings (Hartquist *et al.*, 2003) as well as in cometary tails and low temperature laboratory dusty plasmas (Konopka *et al.*, 2000; Sato *et al.*, 2001) that are nonuniform and magnetized.

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